1-) Multihomogenous Nonnegative Polynomials and Sums of Squares

In 1888 Hilbert classified the cases where every nonnegative polynomial is a sums of squares. In 2006, Greg Blekherman made a quantitative analysis of how often a non-negative degree d form is a sums of squares. He concluded there are significantly more nonnegative polynomials than sums of squares. In this article we generalize Blekherman's work to the case of multihomogenous polynomials. The results in this paper also happen to have interesting implications in quantum information theory.

2-) Probabilistic Condition Number Estimates for Real Polynomials I - II
(with J. Maurice Rojas and Grigoris Paouris)

Condition numbers play a vital role in the design and analysis of numerical algorithms. Probabilistic analysis of condition numbers are meant to model typical instances for such algorithms. Existing literature on probabilistic analysis of condition numbers for real polynomials was restricted in two ways: it did not allow any fixed sparsity pattern and it could only consider Gaussian random polynomials. Our work removes both of these restrictions using tools from geometric functional analysis.

3-) Approximating Nonnegative Polynomials via Spectral Sparsification

We study polyhedral approximations to the cone of nonnegative polynomials. We show that in the case of degree 2d forms with n variables any polyhedral approximation has to have exponentially many facets. This result is based on Gaussian concentration. We also show that for the case of fixed dimensional subspaces (such as the space of symmetric polynomials) a constant ratio polyhedral approximation with polynomially many facets always exist. This result is based on the solution of Kadison-Singer problem. We also provide a randomized construction of this polyhedral approximation utilizing the epsilon-net theorem from computational geometry.

4-) Tropical Varieties for Exponential Sums
(with J. Maurice Rojas and Grigoris Paouris)

We define tropical varieties for polynomials with real exponents and study this object with a computational complexity point of view. We show that the tropical variety provides a cheap approximation for the location of the zero set of an exponential sum. Besides the computational aspects, this paper also has the merit of unifying some classical results in complex analysis with the new perspective introduced by the advent of tropical geometry.

5-) On the Expected Number of Zeros of Random Fewnomials
(with Peter Bürgisser and Josue Tonelli-Cueto)

Descartes' rule of signs shows that the number of real zeros of a real polynomial is controlled by the number of its terms rather than its degree (1636). Khovanski proved a multivariate generalization of Descartes rule with bounds that are exponential with respect to the number of terms (1988). Since then it's an open problem whether the correct upper bound is actually polynomial (all known examples have polynomially many zeros with respect to the number of terms). We show that the expected number of real zeros of a random sparse polynomial system admits a polynomial upper bound. To the best of our knowledge this is the first result for random sparse polynomials that is based only on the number of terms.

6-) Plantinga-Vegter Algorithm Takes Average Polynomial Time
(with Felipe Cucker and Josue Tonelli-Cueto)

The purpose of this paper is to start transferring condition number technology of numerical analysis to computational geometry. Algorithms in computational geometry are usually considered in Real RAM model assuming perfect precision. This proved to be very harmful for geometric modeling problems causing many technical and practical issues. This paper showcases condition numbers and smoothed analysis by focusing on a popular algorithm due to Plantinga and Vegter for meshing real curves and surfaces.

7-) The Rank of Sparse Random Matrices
(with Amin Coja-Oghlan, Samuel Hettereich, H. Maurice Rolvien)

It is well known that nxn random matrices have full rank when they have more than nlogn non-zero entries; this is connected to the emergence of giant component phomena in probabilistic combinatorics. Less is known in the case of sparse random matrices with linear or less non-zero entries. In this paper we prove a rank formula that holds over any field. The proof is based on a combination of algebraic insight with statistical physics techniques. Prior work was based on deliberate combinatorial considerations that hold only over finite fields.

This paper is meant to be the first step of a longer term project with Amin on using real algebraic techniques to study partition functions.

8-) A Polyhedral Homotopy Algorithm for Real Zeros
(with Timo de Wolff)

Polyhedral homotopy algorithms are among the most powerful and fast methods to solve sparse polynomial systems over the complex numbers since early 90's. Over the real numbers however these algorithms do not work due to a simple geometric reason: there is no generic behaviour in real geometry. In other words, there is no fundamental theorem of algebra over the reals. In this article we develop the first polyhedral homotopy method for solving (certain) real polynomial systems. Our algorithm utilizes geometry of A-Discriminants -following the monumental book by Gelfand, Kapranov, Zelevinsky-, Viro's combinatorial patchworking method, some entropy type convex programming, and standard path trackers from numerical analysis.

9-) The Multivariate Schwarz-Zippel Lemma
(with Levent Dogan, Jake Mundo, Elias Tsigaridas)

Imagine a set of M points in the real plane together with a set of N lines. How many incidences can happen between these two sets? A famous theorem by Szemeredi and Trotter answers this question. Now imagine a finite set of points S on the real line, and create a grid S^3 in R^3 by taking the cartesian product of S with itself three times. Now take a polynomial p with 3 variables and degree d. How many zeros can p have on the grid S^3? This question is answered by another famous result called Schwarz-Zippel Lemma. In this paper we provide a common generalization of these two famous results to the multivariate case alongside with applications in combinatorial geometry. The paper also includes an efficient symbolic algorithm to detect pathological polynomials p for which no Schwarz-Zippel type bound holds.